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THE AGGREGATE PRODUCTION FUNCTION AND TECHNICAL CHANGE: THE CANADIAN EXPERIENCE

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled THE AGGREGATE PRODUCTION FUNCTION AND TECHNICAL CHANGE: THE CANADIAN EXPERIENCE submitted by G. Lynne Orman in partial fulfilment of the requirements for the degree of Master of Arts.

ABSTRACT

The fundamental economic goal of sustained economic growth has generated interest in the causes of that growth. This thesis focuses on what has been considered the major contributing factor, technical progress. Solow's two models of technological change, the embodied and the disembodied, are presented and applied to Canadian economic data for the period 1947 to 1964. These results are then analyzed in the context of Canadian economic structure.

The thesis concludes that investment in capital stock, and hence embodied progress, is of prime importance for growth of the Canadian economy. However it is vital that there be concurrent investment in education, research and development in order that the maxium productivity is achieved by the capital stock.

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CHAPTER I

INTRODUCTION

Post-war Studies of Technical Progress

Technical change has become a focal point in the explanation of post World War II economic growth. The publication of Douglas'[1] early investigation into the relationship between capital, labor and output marked the beginning of an upsurge of interest in productivity and technological change. Using the Cobb-Douglas production function he showed that these two inputs (capital and labor) could account for all growth just after the turn of the century. However post-war research revealed that unadjusted capital and labor inputs could not fully explain the growth rate of output. Attention was then focused on a third variable, technological progress.

The first article to recognize that the rate of growth of output exceeded what could be attributed to quantitative growth in capital and labor was published by Abramowitz.[2] His celebrated residual demonstrated that most of the increase in output per capita was unaccounted for by these two inputs. This residual was identified as technical change.

In 1957 Solow specified his disembodied model and concluded that technical progress accounted for 90 per cent of the growth in output.[3] After Solow's article economists set out to explain the residual, to decrease the ignorance associated with increased productivity. The disem-

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[1] C. W. Cobb and P. H. Douglas, "A Theory of Production", American Economic Review, Supplement (March, 1928), pp. 139-165.

[2] Moses Abramowitz, "Resource and Output Trends in the United States Since 1870", Papers and Proceedings of the American Economic Association, vol. XLVI (May, 1956), pp. 5-23.

[3] R. W. Solow, "Technical Change and the Aggregate Production Function", The Review of Economics and Statistics, vol. XXXIX (August, 1957), pp. 312-320.

bodied model suggests that technical progress is not built into capital and labor inputs. Rather, it describes technological advance as a costless process of learning about our environment. Included in disembodied progress are the productivity benefits of increased education and skill attainment, of improved health, of design and product innovations attributable to research, of economies of scale, of improvements in organization of markets, and of managerial efficiency.

More frequently however, technical progress necessitates a change in the actual form of inputs. That is, change must be built into capital and labor in order to become effective. Solow's 1959 publication recognized the embodiment of technical change in capital.[1] He realized that more recently produced capital units differed in size, form and mode of operation from those of earlier vintage. Solow implied that technical change and improvements in the productivity of capital were synonomous.

Denison made an even more detailed analysis.[2] Not only did he allow for improvements in capital, but he adjusted the labor input to allow for changes in quality.

While economists have been busy investigating economic growth in the United States and evolving thesis and anti-thesis as to its nature and source, very little work has been done on the Canadian scene. There are

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[1]R. W. Solow, "Investment and Technical Progress", Mathematical Methods in the Social Sciences, 1959, ed. K. J. Arrow, S. Karlin and P. Suppes (Stanford: Stanford University Press, 1960), pp. 84-104.

[2]Edward Denison, "The Sources of Economic Growth in the United States and the Alternatives Before Us", Supplementary Paper No. 13, Committee for Economic Development, 1962.

two noteworthy exceptions. Output per man-hour was estimated for major economic sectors in Output, Labor and Capital in the Canadian Economy[1] and more recently Lithwick, Post and Rymes estimated the contribution of measured factor inputs to the growth of output in thirteen major manufacturing groups.[2]

This thesis will use the aggregate production function technique to estimate the contribution of technological change to Canadian economic growth between 1947 and 1965. Using the two models developed by Solow (the disembodied and the embodied) we derive an estimate of technical progress in the post-war economy and draw some conclusions as to its sources. However, before proceeding, a brief discussion of the production function and the problems associated with it will be given.

The Production Function and Technical Progress

The aggregate production function has become a popular tool for economists seeking the sources of increased productivity. The production function is an engineering relation between a given set of inputs and the maximum amount of output which they produce. Using the conventional representation of a firm's production function, we have

$$1.1 \qquad x_i = \phi\,(v_{i1}, v_{i2}, \ldots, v_{in})$$

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[1] William C. Hood and Anthony Scott, "Output, Labor and Capital in the Canadian Economy", (Ottawa, 1957).

[2] N.H. Lithwick, George Post and T.K. Rymes, "Postwar Production Relationships in Canada", The Theory and Empirical Analysis of Production, Studies in Income and Wealth, No. 31, National Bureau of Economic Research, (New York, Columbia University Press, 1967), pp. 139-275.

where x_i is the maximal amount of output of commodity i which can be produced by the inputs v_{ij} ($j = 1, 2, \ldots, n$). The function is single valued and has continuous partial derivatives (i.e. well-behaved).

If we consider a commodity, x_k, with only two inputs (v_{k1}, v_{k2}) then the production surface can be represented in three dimensional space by Figure 1a. However a simpler visual aid is obtained by projecting the surface on a two dimensional plane. (Figure 1b) The isoquant x_k^s depicts all combinations of the inputs v_{k1} and v_{k2} which will produce the amount x_k^s of the commodity k. The position of each isoquant is determined by the prevailing technology. If technology changes then the production surface shifts and the isoquant which represented x_k^s moves to, say, x_k^1. Here the quantity of output (k) has not changed, but fewer inputs are required to produce it. The amount of technical change is measured by the prependicular distance between x_k^s and x_k^1.

The efficiency of the technology[1], the degree of economies of scale that are technologically determined[2], the degree of capital intensity of a technology[3] and the ease with which capital is substituted for labor[4] are

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[1]The efficiency of a technology determines the quantitative output that is the product of given inputs.

[2]Economies of scale are defined as increasing, constant or decreasing if, for a given proportional increase in inputs, output is increasing by a greater, constant or smaller proportion respectively.

[3]Capital intensity describes the proportion of capital relative to labor used in the production of a unit of output.

[4]Elasticity of substitution reflects the ease with which one factor may be substituted for another.

characteristics of the production known as an abstract technology.[1] Technological change may be described in terms of these properties.

Only efficient techniques are described by the production function; that is, it depicts only the maximal output obtainable from a given collection of inputs. Technical change may alter the optimal use of the inputs. Economies of scale can also be affected by changes in technology. For example, pre-war tests of the American economy suggested that the system was experiencing constant returns to scale. However recent studies have indicated the existance of increasing returns.[2] Further evidence of technological change is given by a variation in the capital intensity (as reflected by a change in the capital-labor ratio) or a shift in the ease of substitution (as mirrored by a change in the elasticity of substitution).

A well-behaved production function must satisfy a minimal set of economic criteria.[3] These are:

1. The marginal product of output with respect to each input must be positive. A non-positive marginal product would indicate that output could be increased by leaving some input idle.

$$\frac{\delta x_i}{\delta v_{ij}} \geqq 0 \qquad \text{(for all i, j)}$$

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[1] Murray Brown, "On the Theory and Measurement of Technological Change", (Cambridge: Cambridge University Press, 1966) p. 12.

[2] A.A. Walters, "Economies of Scale in the Aggregate Production Function", Discussion Paper, Series A, University of Birmingham, no. XXIX, (April, 1962)

[3] Brown, op cit, p. 29.

2. Each marginal product should decrease as output increases over the relevant range. This is the law of diminishing returns.

$$\frac{\delta^2 x_i}{\delta v_{ij}^2} < 0 \qquad \text{(for all i, j)}$$

Of the many functional forms which satisfy these criteria, the Constant Elasticity of Substitution function and the Cobb-Douglas[1] (which is a special case of the C.E.S.[2]) are the most popular. However both admit the possibility of upward sloping marginal product curves when strong economies of scale are present.

The Aggregation Problem

Thus far we have been considering a microeconomic proposition, the firm's production function. In order to look at the impact of technological change on the economy as a whole the individual production functions must be aggregated. When there are m distinct commodities produced a single index number is chosen to represent the vector of outputs thus;

2.1 $X = (x_1, x_2, \ldots, x_m)$

Similarly each input is aggregated so that the index number V_j represents all the input v_{ij} ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$) used in the

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[1] $Q = AK^{\alpha}L^{\beta}$ where Q is output, K is capital, L is labor, A represents a shift parameter, α and β are the elasticities of output with respect to capital and labor.

[2] $Q = \gamma[\kappa K^{-\alpha} + (1-\kappa)L^{-\alpha}]^{-\frac{v}{\alpha}}$ where γ denotes the efficiency of a technology, K is indicative of capital intensity and v indicates the degree of returns to scale.

production of the m commodities, viz

2.2 $V_j = (v_{1j}, v_{2j}, \ldots, v_{mj})$

Therefore the aggregate production function is given by

2.3 $X = \phi(V_1, V_2, \ldots, V_n)$

Leontief's theorem for aggregation states that it is a necessary and sufficient condition for the grouping of variables that the marginal rates of substitution between any two variables shall be a function only of variables in that group. Therefore it is independent of the value of any variable in any other group. With reference to the production function, $X = \phi(V_1, V_2, \ldots, V_n)$, this implies that the marginal rate of substitution between a unit of V_1 and any other unit of V_1 is constant and independent of the amount of V_k $(k = 2, 3, \ldots, n)$ employed. Whether any production function is able to meet these conditions is debatable.

There has been a flurry of arguments over the validity of the aggregate production function. The problem involves the relationship between micro and macro functions under aggregation. Ideally the production function is a technical relationship between physical units of input and the maximum amount of output that they can obtain. This concept is quite clear at the firm level where, presumably, engineers design their systems to achieve the maximum output from given inputs. At the macro level, however, the real meaning of the production function becomes hazy. In order to aggregate the output of several firms single indices are used to represent the vectors of input and output. This pre-

sents the difficulty of "adding up" heterogeneous units of capital, labor and output. Further, the concept of a production technique which was built into the individual micro relations becomes a complex notion at the aggregate level.

The aggregation of heterogeneous units of capital has been one of the prime focal points of the aggregation argument. Capital stock consists of buildings, machinery and equipment. Aggregation of such a complex variable introduces numerous measurement problems. In fact such an aggregation implies that one unit of capital is a perfect substitute for another. Joan Robinson, Solow's antagonist in the argument, has vehemently protested against the use of a simple factor production function.[1] She has argued that it is ridiculous that the elasticity of substitution between a unit of capital and any other unit of capital should be so high. However Samuelson has shown that heterogeneous models can be treated as if they came from a simple production function using a surrogate capital variable.[2] In addition to this problem, the indices used to deflate the capital series reflect the cost of producing capital rather than the value of the capital services. Walters points out that this results in underestimation of the capital stock since the increased

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[1] Joan Robinson, "The Production Function and the Theory of Capital", Review of Economic Studies, vol. XXI.

[2] Paul A. Samuelson, "Parable and Realism in Capital Theory: The Surrogate Production Function", Review of Economic Studies, vol. XXIX (June, 1962), pp. 193-207.

productivity of inputs in the capital goods industry is ignored.[1] Further ambiguities in capital aggregation arise because of the measurement of depreciation. Usual methods of depreciation specify a single rate which is applied to all forms of capital. This ignores the obvious differences in life spans of various forms of machinery and structures.

The problems of aggregation are not unique to the production function. They are inherent to any type of macro analysis (national econometric models, input-output tables, international productivity comparisons, etc.). Economists seem to have taken the position that "this kind of aggregate economics appeals or it doesn't".[2] Knowledge of the relationship between inputs and outputs is vital for growth policy. Given this need and the lack of a superior tool, they have relied upon the aggregate production to indicate the sources of growth and to aid policy decision making.

The Cobb Douglas Production Function

Probably the most popular production function is the Cobb-Douglas function. This function represents output as a function of only two inputs, capital and labor. The contributions of all other inputs are lumped together, along with an assortment of other ills, into a shift parameter. This shift parameter has often been identified with tech-

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[1] A.A. Walters, "Production and Cost Functions, An Econometric Survey", Econometrica, vol. XXXI (January - April, 1963), pp. 11-14.

[2] R.W. Solow, "Technical Change and the Aggregate Production Function", p. 312.

nological change. Using a specialized version of the Cobb-Douglas, the complete model for estimation of the production function may be represented thus;

$$2.4 \quad Q(t) = A(t)L(t)^{\alpha_L} K(t)^{\alpha_K}$$

$$2.5 \quad \alpha_L + \alpha_K = 1$$

$$2.6 \quad \frac{\delta Q\ L}{\delta L\ Q} = \alpha_L = w_L$$

$$2.7 \quad \frac{\delta Q\ K}{\delta K\ Q} = \alpha_K = w_K$$

where Q is output, A is technical change, L is labor, K is capital, α_L and w_L are the elasticity of output with respect to labor and the relative share of labor respectively, and α_K and w_K are the elasticity of output with respect to capital and the relative share of capital respectively.

Equation 2.4 is the Cobb-Douglas production. The homogeneity condition is specified in 2.5. It dictates constant returns to scale. Equations 2.6 and 2.7 specify producer equilibrium under conditions of perfect competition. That is, the marginal physical product of output with respect to each input (e.g. $\frac{\delta Q}{\delta L}$) is equal to the ratio of the input price to the output price.

In applications of the production function to be considered here, equations 2.5, 2.6 and 2.7 are taken as identities. Equation 2.4 is estimated in the form

$$2.8 \quad \log Q(t) = \log A(t) + w_K \log K$$

A survey of time series regression studies of this form reveals that the homogeneity condition is approximately true and that the marginal

productivity of labor is equal to the wage rate.[1] The value of these conclusions is dependent on the estimates being good approximations of the structural parameters. However, none of these studies checked for simultaneous equation bias or identification.

The Identification Problem

The "impossibility theorem" developed by Diamond and McFadden questions the ability to identify the function under conditions of technical change.[2] It states that if a time series is applied to a neo-classical production function it is impossible to identify the function. An alternative function having an arbitrary bias or arbitrary elasticity at these points could generate the same data. Nerlove has noted that the "absence of non-factor augmenting change or the assumption of exponential factor augmentation change is sufficient for identification".[3] Variation in the factor shares and in the capital-labor ratio is also essential for identification.[4]

Identification becomes even more difficult in the face of the confluence property of production functions which are estimated through the use of time series. Capital, labor and output exhibit constant growth

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[1]Walters, loc. cit.

[2]P.A. Diamond, "Technical Change and the Measurement of Capital and Income", Review of Economic Studies, vol. XXXII (October, 1965), p. 289.

[3]Marc Nerlove, "Estimation and Identification of Cobb-Douglas Production Functions", (Chicago, Rand McNally and Company, 1965), p. 97.

[4]Ibid., p. 98.

trends which probably do not diverge enough to permit observation of separate trends. It has been argued that this results in estimation of "relations between historical rates of growth of labor, capital and output but the coefficients that do this do not measure marginal productivity".[1] Brown has suggested that two refinements decrease the impact of the confluence trait of the three time series; first by adjusting the available capital series into one of utilized capital and second by including time as an exogenous variable.[2] Further he suggests that the significance of the estimates is not invalidated by collinearity when there is a good fit.

An Outline

In Chapter Two Solow's disembodied model is presented and applied to the Canadian economy. Using the new estimates of capital stock for the private sector derived by Evans, the rate of disembodied progress in the Canadian economy was found to be 1.192. This is considerably less than the 1.5 per cent per year obtained by Solow for the American economy for the period 1909 to 1949.

The embodied model is the subject of Chapter Three. Following the pattern of Chapter Two, it was applied to the Canadian economy. The estimated value of λ is .040. This not only exceeds the value yielded by

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[1] E.H. Phelps Brown, "The Meaning of the Fitted Cobb-Douglas Production Function", Quarterly Journal of Economics, vol. LXXI (November, 1957), p. 555.

[2] Murray Brown and Joel Popkin, "A Measure of Technological Change and Returns to Scale", Review of Economics and Statistics, vol. XLIV (1962), p. 409.

the disembodied model, but is also larger than that obtained by Solow for the American economy (.025).

In Chapter Four a brief survey of refinements to the embodied model is presented. Finally the implications for model building of non-neutral change are discussed.

The results are summarized in Chapter Five and their policy implications noted.

CHAPTER II

THE DISEMBODIED MODEL

The Disembodied Model

In his famous article of 1957, Solow demonstrated the importance of technical progress as opposed to growth in capital per head as the instigator of increased output per head.[1] Since 1957 numerous economists have been fascinated by the contribution of technical change to growth.

The basic model may be represented thus;

$$3.1 \quad Q(t) = A(t)\ f\left[K(t),\ L(t)\right]$$

where Q, K and L represent aggregate output, inputs of capital and labor respectively at time t. A(t) is a shift factor which is meant to be an approximation of neutral technical change. Change is specified as neutral in order to separate the effects of technological progress on output per unit of labor from the effects of increased capital intensity. A(t) incorporates all the elements influencing output, other than capital and labor, without distinguishing between their separate effects.

This representation of the production function (3.1) requires that technical progress be Hicksian neutral. This permits the identification of technological change with shifts in the production function. Hicksian neutral change affects the labor and capital inputs equally. It is neither labor-saving nor capital-saving. When factor proportions are held constant, the marginal rate of substitution of capital for labor is unaffected. Neutral change can be produced by variations in the efficiency of a tech-

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[1] Solow, "Technical Change and the Aggregate Production Function".

nology or alterations in the returns to scale. This can be represented by a two factor graph of a family of isoquants (Figure 2a). Q is the output produced under the old technology and Q' is the output produced under the new. Along Q' more output is produced with the same level of inputs. Neutral progress merely alters the scale of the axis. Conversely a non-neutral change can be either capital-saving (labor-using) or labor-saving (capital-using). This is produced by variations in the capital intensity and/or in the ease of substitution. Therefore the isoquant pivots as shown in Fibure 2b.

The Hicksian definition of neutral technical change can be contrasted with Harrod's proposition.[1] Change is neutral in the Harrod sense if the average product of capital is independent of technical change when the marginal product is constant. In Harrod's definition technical progress is purely capital augmenting. This complies with his concept that labor is the primary factor of production whereas capital is produced. Harrod neutral change therefore has a bias in favor of labor. Since it does not affect capital and labor equally it contradicts the Hicksian definition. Harrod neutral change does not permit the use of the multiplicative feature of Solow's model.

Solow's production function is homogeneous of degree one. Since all inputs are classified as either capital or labor, this implies that all

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[1]R. Harrod, "The Neutrality of Improvements", The Economic Journal, vol. LXXI (June, 1961).

factors are paid the marginal products (i.e. Euler's theorem under conditions of constant returns to scale).

Taking the total differential of 3.1 with respect to time and dividing by Q, Solow arrives at

$$3.2 \quad \frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + A\frac{\delta f}{\delta K}\frac{\dot{K}}{Q} + A\frac{\delta f}{\delta L}\frac{\dot{L}}{Q}$$

where the dots represent time derivatives (e.g. $\dot{Q} = \frac{dQ}{dt}$). Now $\frac{\delta Q}{\delta K} = A\frac{\delta f}{\delta K}$ and $\frac{\delta Q}{\delta L} = A\frac{\delta f}{\delta l}$. Substitution in 3.2 for $\frac{\delta f}{\delta L}$ yields

$$3.3 \quad \frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + \frac{\delta Q}{\delta K}\frac{\dot{K}}{Q} + \frac{\delta Q}{\delta L}\frac{\dot{L}}{Q}$$

which can also be written as

$$3.4 \quad \frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + \frac{\delta Q K \dot{K}}{\delta K Q K} + \frac{\delta Q L \dot{L}}{\delta L Q L}$$

In equilibrium under perfectly competitive conditions $\frac{\delta Q}{\delta K}$ (the marginal physical product of output with respect to capital) is equal to the ratio of the price of capital to the price of output. Therefore $\frac{\delta Q}{\delta K}\frac{K}{Q}$ represents the relative share of capital in output (w_K). Similarly $\frac{\delta Q}{\delta L}\frac{L}{Q} = w_L$. Substitution yields

$$3.5 \quad \frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + \frac{w_K\dot{K}}{K} + \frac{w_L\dot{L}}{L}$$

Since all output is divided between capital and labor $w_L + w_K = 1$.

Therefore

$$3.6 \quad \frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + w_K\frac{\dot{K}}{K} + (1 - w_K)\frac{\dot{L}}{L}$$

Let $q = \frac{Q}{L}$, then $\dot{q} = \frac{\dot{Q}}{L} - \frac{Q\dot{L}}{L^2}$ and $\frac{\dot{q}}{q} = \frac{\dot{Q}}{Q} - \frac{\dot{L}}{L}$. Similarly $k = \frac{K}{L}$ and $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$. This gives

3.7 $$\frac{\dot{q}}{q} = \frac{\dot{A}}{A} + w_K \frac{\dot{k}}{k}$$

Using 3.7 to find $\frac{\dot{A}}{A}$, only series of output, capital and the share of capital are needed.

How much of the increase in post World War II output in Canada is due to technological change and how much is due to increased capital? To answer this question I have applied Solow's model to Canadian data expressed in constant 1957 dollars covering the period 1947 to 1964.

The Data

The output series (q) measures real private non-agricultural gross national product per non-agricultural man-hour. Exclusion of the government sector eliminates the conceptual problems of measuring the real output of the private sector. Agriculture has been eliminated in order to conform more closely to the homogeneity condition. This decision reflects the belief that agriculture is experiencing decreasing returns to scale.

The man-hours data was constructed from two confidential series supplied by the Dominion Bureau of Statistics. Using the Databank tape at the Bank of Canada the series of number of paid workers in the private non-agricultural sector was combined with the series for the total number of hours worked per worker in the private non-agricultural

sector to arrive at a series for man-hours.

The capital stock series is derived from the work of Evans who constructed a quarterly private capital stock series as part of his investment sector in the Bank of Canada's aggregate model. His series was calculated using second quarter 1949 benchmark values, a series of quarterly investment flows and an exponential rate of decay. Capital stock was separated into non-residential construction and machinery and equipment. Rymes' study provided the benchmark for manufacturing stock at the end of the second quarter 1949. Estimates of most other non-manufacturing industries were obtained from the D. B. S.. The remaining sectors; "Finance, Insurance and Real Estate" and "Commercial Services" were imputed from the estimates of Hood and Scott. In order to reconcile the capital stock series with the output series described above, Evans' net capital stock figures were adjusted to exclude agriculture. The benchmark figures are as follows;

Net Capital Stock (2nd Quarter 1949)
(millions of 1957 dollars)

	Non-residential Construction	Machinery and Equipment
Manufacturing	3,785	2,963
Forestry	148	74
Fishing and Trapping	8	94
Mining, Quarrying & Oil Wells	557	251
Construction	79	255
Transport, Storage & Communication	4,342	1,809
Public Utilities	2,314	680
Trade	1,337	377
Finance, etc. & Commercial Services	1,089	345
	13,659	6,848

Using a depreciation rate of .05 for machinery and equipment and .01 for non-residential construction the series were pushed back to the first quarter of 1947 and forward to the last quarter of 1964. The investment series used for these operations were the National Accounts' Gross Investment on New Machinery and Equipment and Gross Investment on New Non-Residential Construction. Both of these were adjusted to exclude agriculture by using the Investment in Agriculture series in White. At this point we had a quarterly series for net capital stock in machinery and equipment and a similar series for non-residential construction. Using a subroutine built into the Bank of Canada's massager program these were converted to annual data and then summed to yield a series of private net capital stock 1947-1964. This series measures capital in place. In order to arrive at an estimate of capital in use, which is the appropriate variable for the production function, it was assumed that capital and labor are employed in the same proportions. The rate of labor employment was obtained by dividing the figure for the total civilian labor force employed by the total civilian labor force. This employment rate was then used to deflate the net capital stock series. Net utilized capital stock per man-hour was arrived at by dividing the latter series by total paid man-hours.

The share of capital series was constructed from the D. B. S. tables on Nation Income and Gross Expenditure. It includes corporation profits before taxes, rent, interest and miscellaneous investment income, capital consumption allowances and fifty per cent of net income of non-farm unincorporated enterprise. Lithwick, Post and Rymes have used a 50:50

split of net unincorporated business rather than the 35:65 division adhered to by Solow.[1]

The Empirical Results

The results are shown on Table IV. Over the eighteen year period there was an average upward shift of 1.912 per cent per year. (The time path of A(t) is shown in Figure 3.)

This value is somewhat less than Solow's 1.5 per cent per year. There are several plausible explanations for this. The first to be considered is the base year of the two studies. If we set A(1945) = 1.000 in Solow's study his series for A(t) is markedly lower (e.g. A(1946) = .983, A(1947) = .999, A(1948) = 1.023). Because there would be so little capital in place in 1909 any additions to capital stock would have a far greater impact on growth than similar increments in 1947. This observation may be generalized to include technological change. That is, a given measure of technological change would have a greater impact in 1909 than in 1947.

A further explanation derives from the fact that technical change is a residual and much of the post-war growth in Canadian output can be accounted for by growth in inputs, particularly capital. Between 1947 and 1965 Canadian investment as a percentage of GNP exceeded that of any of the major western countries.[2] This is partially explained by the resource

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[1]A 35:65 split was tried, however the impact on technological change was negligible.

[2]Derek A. White, "Business Investment to 1970", Staff Study No. 5 prepared for the Economic Council of Canada, p. 15.

oriented nature of the Canadian economy.[1] White has noted that resource industries, with the exception of forestry require a far larger capital stock for a given value of output than the balance of the economy. The influence of capital inputs can be seen by manipulating our results. The private non-agricultural GNP per man-hour, net of technical advance, was $2,879.42 million in 1965. Between 1947 and 1965 it increased by $957.41 million ($3,432.27 - $2,494.86). Over the same period private non-agricultural GNP increased by $957.41 million to $3,432.27 million. If we net out technological advance, the corresponding figure for 1965 was only $2,879.42. Therefore we may conclude that $404.56 million of the $957.41 million (or approximately 42%) of the increase can be attributed to increased capital intensity. This compares to an American figure of 11%. Support for these results may be drawn from investment data. Investment in Canada amounted to an impressive 22.9 per cent of GNP between 1949 and 1963 as compared to an average of approximately 13.1 per cent between 1926 and 1949.[2,3]

The model is fairly insensitive to data changes. This was verified by an application to another data set which incorporated three alterations. A man-years series was used in lieu of man-hours data, agriculture was included throughout and the capital stock series was replaced by one structured by a 1946 base estimate and the D.B.S. series Business Gross

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[1] Ibid., p. 30.

[2] Ibid., p. 27.

[3] Ibid., p. 28.

Fixed Capital Formation. The resulting figure for technological advance was 1.142 per cent. This is not drastically different from the 1.192 figure reported above. Most of the difference would seem to be explained by a slower growing capital series in the latter. Although it is difficult to make precise inferences about the contribution of agriculture, the inclusion of this sector tends to reduce our figure for technological advance. This agrees with hypotheses made elsewhere. Lithwick, Post and Rymes found that agricultural productivity is low relative to other industries.[1]

A Critique

One should note some of the difficulties associated with Solow's measure. The production function has been assumed to be linear homogeneous. That is the elasticity of output with respect to capital and the elasticity of output with respect to labor must add to one. As mentioned above, the Canadian economy is resource oriented and these industries are highly capital intensive. The individual fixed capital items required in drilling, mining, etc. are frequently very expensive and their cost must be incurred before extraction begins. These costs can only be offset by large scale operations. Further, the ratio of fixed to variable costs is very high. Therefore average cost can be expected to decrease over a wide range of output. For example, it has been shown that petroleum is a decreasing cost industry.[2] Diwan has demonstrated that if the production function is actually homogeneous of degree greater than one

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[1] Lithwick, Post, Rymes, loc. cit., p. 197.

[2] "The Price of Oil in Western Europe", prepared by the Secretariat, Economic Commission for Europe, E/ECE 1205 (Geneva, 1955) pp. 16-18.

then any estimates of technological change using the linearity assumption will be biased upward.[1] Whether the Canadian economy is experiencing economies of scale is debatable. Prewar studies indicated linearity. No recent evidence is available. If we use American experience as a yardstick then the fact that Walters has found increasing returns in the United States for the forty-one years 1909-1949[2] is at least suggestive of similar conditions in Canada.

The upward bias may be offset by several other factors. As Nelson has suggested, technological advance acts as a catalyst in the growth process thereby inducing increases in other factors.[3] With advancing technological knowledge new and more productive techniques requiring new physical capital become available. This contribution is ignored in Solow's model.

Some have found fault with Solow's assumption of neutral technological change. In his 1957 article he attempted to verify this assumption by a scatter of proportional changes in the function against the capital-labor ratio. He found no relationship, thereby supporting his hypothesis that shifts were neutral on the average. This assertion does not imply that

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[1] Romesh K. Diwan, "Productivity in Australian Manufacturing", a paper presented at the second Far Eastern Meeting of the Econometric Society, 1967, p. 5.

[2] A.A. Walters, "A Note on Economies of Scale", Review of Economics and Statistics, vol. XLV (June, 1963) pp. 425-427.

[3] Richard R. Nelson, "Aggregate Production Functions and Medium Range Growth Projections", American Economic Review, vol. LIV (September, 1964), pp. 575-605.

each shift was neutral. The capital-labor ratio can change in such a way as to allow proportional changes in the function to be zero and still permit non-neutral technical change. Rather, this test was an attempt to show that there was no persistent bias in the shifts. The existence of non-neutral change makes the application of the Solow model a formidable exercise.

In order to arrive at an estimate of capital utilization, the capital series was corrected by assuming that capital and labor are employed in the same degree. To the extent that employers hesitate to discharge skilled labor when output is decreasing[1] or to the extent that employers lay off proportionally more labor than capital to take advantage of existing capital, it is possible that such a series would deviate from actual utilization. Resek has shown that a capital stock series adjusted by unemployment tends to lag behind the cycle for actual utilization.[2] If we accept this conclusion then the Solow method underestimates capital stock in the downswing and hence over-estimates technical change.

Alternate measures of capacity utilization have been developed.[3] One such measure is the Wharton School Index. In this measure an index

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[1] The Annual Report of the Governor of the Bank of Canada to the Minister of Finance for the year 1967 suggests that Canada's poor productivity performance in 1966 and 1967 "may have been affected by a reluctance on the part of employers to risk losing trained workers who have been acquired during the earlier period of labour shortages". (p. 28)

[2] R. W. Resek, "Neutrality of Technological Progress", The Review of Economics and Statistics, vol. XLV (February, 1963), pp. 55-63.

[3] Almarin Phillips, "An Appraisal of Measures of Capacity", American Economic Review, Papers and Proceedings, vol. LIII (May, 1963) pp. 275-293.

of production is plotted against time. The peaks in the line graph are defined as capacity output. A line is extrapolated from one peak to the next peak which is higher than the starting peak. This trend line is defined as capacity output for the intervening period. Percentages utilization is then measured by the vertical distance between the capacity trend line and the plotted data.

The major criticism of the Wharton Index is that the peaks may not actually represent capacity output. Capacity may in fact fall above or below this level. Further criticism points to the fact that capital formation is most intense during the upturn of the cycle and is not evenly distributed over the cycle as suggested by the index. While the Wharton Index offers a quick and easy method of measuring capacity utilization it is impossible to use it for Canadian data between 1947 and 1966. Over this period output grew rather steadily with the exception of 1957 when the economy suffered a slight set-back. A time graph of output would not reveal any peaks.

The McGraw-Hill measure of capacity appraised by Phillips involves surveys of business intentions with respect to utilization. Such surveys have not been undertaken in Canada. Other capacity utilization measures make use of capital output ratios. However the estimation of constant dollar book value of fixed capital, which is required for the index is a monumental task in itself.

The disembodied model implies that growth is virtually independent of investment patterns. The function of gross investment as a vehicle for

carrying new technology into place is ignored. Indeed, the disembodied view suggests a very limited role for gross investment. If capital and labor are held constant, output will continue to increase through increased efficiency or improvements in returns to scale. This implies that the production function will shift outward as a given technical epoch matures. Labor becomes more skillful and management becomes more efficient as the same tasks are repeated. Alternately the production function may shift because of the introduction of new techniques. However, since labor and capital input are held constant, this implies an unusual innovation which changes the organization of markets and/or production but does not change the proportion of labor to capital. The disembodied view further limits the role of investment since the impact of modernization is ignored. In order to recognize the embodiment of technical progress, Solow has developed a new production function which focuses on the age distribution of capital. It is to this mode that I now turn.

CHAPTER III

THE EMBODIED MODEL

Introduction

The contribution of additional capital which is technologically fixed is highly dependent on the rate of technical advance. However, there are limits to the increases in productivity that can be achieved by increasing the capital stock with additions which are technologically static. As labor becomes familiar with the technology existing in capital, increases in output per worker which can be classified as disembodied will be exhausted. Solow's embodied model attempts to account for the fact that most technical change must be embodied in new capital to be effective. Therefore, investment modernizes and deepens the capital stock and allows for the introduction of more disembodied change.

If we take a unit of capital produced in year zero as our standard unit, then a unit of capital produced in the year r, which is a perfect substitute for γ_r standard units symbolizes γ_r efficiency units. The series γ_r , (r=0, 1, 2, . . .), is an increasing one.

The Embodied Model

In his new model Solow assumes that technology is carried into place only through gross investment in capital (i. e. purely capital augmenting). This meets the conditions of a theorem set out by Fisher which states that, in conditions of constant returns to scale, an aggregate capital stock exists if and only if all technical change is capital augmenting.[1]

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[1]F. M. Fisher, "Embodied Technical Change and the Existence of an Aggregate Capital Stock", Review of Economic Studies, Vol. XXXIII, (October, 1965), p. 268.

Solow ignores the disembodied form of capital. The embodied model concentrates on K (t), the number of capital units produced at time v and still in existence at time t. In this model Solow explicitly assumes a Cobb-Douglas production function. Accordingly the output produced at time t by vintage v capital is given by

$$4.1 \quad Q_v(t) = Be^{\lambda v} L_v(t)^{\alpha} K_v(t)^{1-\alpha}$$

where $Be^{\lambda v}$ is an exponential approximation of technology affecting only new capital goods and $L_v(t)$ is the quantity of labor working with $K_v(t)$. Since progress is neutral (the embodied model complies with both definitions of neutral change), the elasticity parameter (α) is the same for all vintages.

It follows that $K_v(t)$ is gross investment in period v. Using a constant mortality rate δ , and letting I(v) represent gross investment, we have

$$4.2 \quad K_v(t) = K_v(v)\, e^{-\delta(t-v)}$$

$$= I(v)e^{-\delta(t-v)}$$

Therefore

$$4.3 \quad Q_v(t) = Be^{\lambda v} L_v(t)^{\alpha} I(v)^{1-\alpha} e^{-\delta(t-v)(1-\alpha)}$$

Under conditions of perfect competition the marginal productivity of labor at time t, m(t), is equalized in all uses. This condition gives.

$$4.4 \quad m(t) = \frac{\delta Q_v(t)}{\delta L_v(t)}$$

$$= \alpha Be^{\lambda v} L_v(t)^{\alpha-1} I(v)^{1-\alpha} e^{-\delta(t-v)(1-\alpha)}$$

Solving this for $L_v(t)$ yields

$$4.5 \quad L_v(t) = \left[\frac{m(t)e^{\delta(t-v)(1-\alpha)}}{\alpha Be^{\lambda v} I(v)^{1-\alpha}}\right]^{\frac{1}{\alpha-1}}$$

If we rearrange this and substitute $e^{\gamma} = \frac{\lambda}{1-\alpha} + \delta$ we get

$$L_v(t) = m(t)^{\frac{1}{\alpha-1}} (\alpha B)^{\frac{1}{1-\alpha}} e^{-\delta t} e^{\gamma v} I(v)$$

or

$$4.6 \quad L_v(t) = h(t)\, e^{\gamma v} I(v)$$

Substituting 3.2 and 4.6 into 4.1 yields

$$Q_v(t) = Be^{\lambda v} h(t)^{\alpha} e^{-\alpha\gamma v} I(v)^{\alpha} I(v)^{1-\alpha} e^{-\delta(t-v)(1-\alpha)}$$

$$4.7 \quad = Be^{-\delta(1-\alpha)t} h(t)^{\alpha} I(v)\, e^{\gamma v}$$

Since Q(t) may be found by integrating over all vintages, v, and since integration of 4.6 gives

$$h(t) = \frac{L(t)}{\int e^{\gamma v} I(v)\, dv}$$

we have Solow's embodied model

$$4.8 \quad Q(t) = Be^{-\delta(1-\alpha)t} L(t)^{\alpha} J(t)^{1-\alpha}$$

where

$$4.9 \quad J(t) = \int_{-\infty}^{t} e^{\gamma v} I(v)\, dv$$

J(t) may be considered a measure of the productivity adjusted capital stock. The factor $e^{\gamma v}$ weights each vintage of capital goods for productivity improvement.

In his 1959 article Solow estimated the rate of productivity advance of vintage capital through the use of extraneous estimates for δ and α. He defines R(t) as $Q(t)^{\frac{1}{1-\alpha}} \div L(t)^{\frac{\alpha}{1-\alpha}}$. With this definition he is able to show that

$$4.10 \quad \frac{\frac{dR}{dt} + \delta R}{I(t)} = B^{\frac{1}{1-\alpha}} \; e^{\frac{\lambda t}{1-\alpha}}$$

$$\therefore \; \text{Log}\left[\frac{\frac{dR}{dt} + \delta R}{I(t)}\right] = \frac{1}{1-\alpha} \log B + \frac{\lambda}{1-\alpha} t$$

It is now possible to get an estimate of $\frac{\lambda}{1-\alpha}$ (the slope coefficient of the linear regression of the logarithm on time) by using time series of Q, L and I.

An Empirical Application

Solow estimated λ for $\alpha = 3/4$ and $\alpha = 2/3$. He found that $\alpha = 3/4$ gave the best fit and a value for λ equal to .025 for the period 1919 to 1953 in the United States.[1]

We have followed Solow's procedure to estimate λ for the Canadian economy between 1947 and 1965. The time series used are basically the same as those used in the disembodied model. Q constitutes Private Non-agricultural Real Gross National Product, L is the employed labor force in man hours and I is the Real Private Gross Fixed Capital Formation. (See Table V).

Two regressions were run using $\alpha = 2/3$ and $\alpha = 3/4$. When $\alpha = 3/4$ the variable t explains but 42.80 per cent of the composite variable of equation 4.10 and is significant at the 95% level. When $\alpha = 2/3$ only 34.03 per cent in explained. This compares with the 72 per cent obtained

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[1] R.M. Solow, Investment and Technical Progress, p. 95.

by Solow.[1] However, the presence of autocorrelation, which is common to most economic data, affects the sampling variances. This invalidates the usual formulae for $\bar{R}^2$.

The values obtained for the improvement factor are significantly above those derived for the disembodied model and slightly above Solow's figure. If $\alpha = 2/3$ then $\frac{\lambda}{1-\alpha} = 4.918 \times 10^{-2}$ which gives an improvement factor of .039.[2] When $\alpha = 3/4$, $\frac{\lambda}{1-\alpha} = 6.830 \times 10^{-2}$. The improvement factor is .040.[3]

The form of Solow's estimating equation is most confusing. Examination 4.10 reveals that the left hand side represents a weighted first difference of productivity per unit of investment.[4] t is designed to represent embodied

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[1] Ibid

[2] $$\text{Log}\left[\frac{R-R_{t-1}}{R} + 0.04R/I\right] = 7.64 + .04918\ t$$

$$\frac{\lambda}{1-2/3} = 4.918 \times 10^{-2}, \quad \lambda = 0.0169 \text{ and } e^{\lambda} = 1.039$$

[3] $$\text{Log}\left[\frac{B-B_{t-1}}{B} + 0.04\ B/I\right] = 11.9098 + .06830\ t$$

$$\frac{\lambda}{1-3/4} = 6.930 \times 10^{-2}, \quad \lambda = 0.1707, \quad e^{\lambda} = 1.040$$

[4] $$\frac{\left[\frac{Q(t)}{L(t)}\right]^{\frac{1}{1-\alpha}} - \left[\frac{Q(t-1)}{L(t-1)}\alpha\right]^{\frac{1}{1-\alpha}} + \delta\left[\frac{Q(t)}{L(t)}\right]^{\frac{1}{1-\alpha}}}{I(t)}$$

$$= \frac{\left[\frac{Q(t)}{L(t)}\right]^{\frac{1}{1-\alpha}}(1+\delta) - \left[\frac{Q(t-1)}{L(t-1)}\right]^{\frac{1}{1-\alpha}}}{I(t)}$$

technological change. However productivity per unit of investment would seem to be an ideal proxy variable for embodied technological change. Equation 4.10 then appears to be a regression of embodied technical progress on time. The low $\bar{R}^2$ indicates that time is unable to explain most of the variation in embodied change. Given that Canada imports innovation from the United States rather than developing it domestically this is not a surprising result. Time would not systematically explain embodied change. Rather, time has a sporadic influence; new techniques and new equipment are perfected elsewhere before Canada adopts them. Therefore, change exhibits a lumpiness instead of a continuous relationship with time.

If Solow's equation is merely a juggling act in order to arrive at a value for λ (and this would appear to be the case) then R^2 has little or no significance. As long as the values for the regression coefficients are unbiased then the value for λ can be accepted with the degree of confidence determined by the t test. Once again the possibility of autocorrelation interferes with statistical criterion. Conventional representation of the t test is incorrect in its presence.

A Critique

The existence of an embodied improvement factor which is greater than the disembodied factor agrees with the hypothesis of Chapter II. That is, growth in technologically improved capital stock has accounted for a major proportion of Canada's output increases. The relatively high improvement factor reflects the Canadian policy of importing machinery and equipment

with the latest improvements from the United States rather than being constrained to develop the advances separately. This conclusion is supported by Lithwick, Post and Rymes who note that the similarity in rates of technical progress support the assertion that Canada has been able to borrow capital, complete with new ideas and techniques,from the United States. However, they dispute the suggestion that embodied technical advance has been important to growth. This conclusion is reached by regressing changes in output in period (s+1) against investment in period s. Rather, they suggest, output increases in Canada are a result of intersectoral movements of labor, in particular the movement of labor out of the relatively lower productivity agricultural industry. This is certainly plausible in the light of Massels work on nineteen industrial groups within U.S. manufacturing between 1946 and 1957. He found that approximately one third of technological change accrued to interindustry shifts of resources. This dynamic process had three causal factors, readjustment to a disequilibrium, adjustment to a continuously shifting equilibrium and indications of interindustry differences in labor quality. In light of the suggestions by Lithwick, Post and Rymes it would be particularly interesting to apply Massell's method to the Canadian scene. However, like so many other interesting studies, this will have to await an industry by industry equivalent stock of capital series.

The assertion that intersectoral labor movements are important to growth need not completely contradict our embodiment argument. The industries with the highest productivity levels have tended to be those with greater capital intensity (e.g. transportation equipment, nonferrous metal

products and electrical apparatus, nonmetallic mineral products and products of petroleum and coal). Presumably labor movements to areas of higher productivity will be directed at these industries.

Other Applications of the Embodied Model

In a later article Solow calculated an equivalent stock of capital series using alternative trial values of the improvement factor, for plant and equipment.[1] From this he was able to use his model to derive estimates of the elasticity of output with respect to the equivalent stock of capital and hence a completely specified production function. This production function was then used to estimate investment requirements for alternate rates of growth of potential output. These results were far more encouraging for investment policy than those obtained from the disembodied model. Investment amounting to twelve to fourteen per cent of GNP would be required to increase the growth rate to four and one-half per cent in the embodied model whereas almost twenty per cent of potential output is required for a four per cent growth rate in the disembodied model.[2]

While the implications for investment are very encouraging in the new model, Phelps has noted that the modernizing effects of expanded investment are limited.[3] The impact of a one shot increase in investment will not last. In order to maintain a higher average age, the investment

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[1] R. W. Solow, "Technical Progress, Capital Formation and Economic Growth", American Economic Review, Papers and Proceedings, Vol. LIT, (May, 1962), pp. 76-86.

[2] Ibid, p. 85.

[3] Edmund S. Phelps, "The New View of Investment: A Neoclassical Analysis", Quarterly Journal of Economics, Vol. LXXVI, (November, 1962), pp. 548-567.

ratio must be a continuously increasing function of time. Since this is not possible the average age of capital will eventually increase to its equilibrium age.

More recently, Solow has attempted to use the Constant Elasticity of Substitution Function, which he developed in co-operation with Arrow, Chenery and Minhas, in order to demonstrate the powers of his embodied model.[1] The elasticity of substitution is invariant with respect to changes in factor prices and relative factor inputs. However, the CES function does not dictate unitary elasticity.

Assuming homogeneity of the first degree, the CES production function is given by

$$3.11 \quad Q = \gamma \left[\kappa K^{-\rho} + (1-\kappa) L^{-\rho} \right]^{-\frac{1}{\rho}}$$

γ is a scale parameter representing the efficiency technology, κ is a measure of capital intensity and σ (the elasticity of substitution of capital for labor) is given by $\frac{1}{1+\rho}$.

Since we are assuming constant returns to scale, neutral technological change is depicted by changes in the parameter γ.

Solow has used both cross-sectional and time series data for two digit manufacturing industries in order to arrive at his estimates for technological progress. Under specified conditions an estimate of σ can be obtained by re-

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[1]R.M. Solow, "Capital, Labor and Income in Manufacturing", Studies in Income and Wealth, National Bureau of Economic Research (Princeton: Princeton University Press, 1961).

gressing $\log \frac{Q}{L}$ on W. Assuming that his production function is applicable to nine regions in the United States, Solow has estimated the substitution parameter ν (and hence ρ) from cross-sectional data. He then uses this extraneous estimator in addition to a time series of the same aggregated industries in order to estimate annual rates of technological progress in each industry.

Solow's estimates of technical change using the CES function are somewhat larger than Cobb-Douglas estimates. Part of the explanation derives from the fact that he estimates purely capital augmenting change rather than neutral change. The parameter γ estimates neutral change only when $\nu = 1$. Therefore, his estimate includes some capital-saving progress in addition to the neutral variety.

The use of the CES function is relatively complex. Nelson has been able to show that with reasonable values of the elasticity of substitution, increases in the capital-labor ratio explain only a small portion of post-war productivity increases.[1] If his conclusion is acceptable then one may use the simpler Cobb-Douglas to analyze growth. Nerlove is opposed to this proposition.[2] He argues that the value of the elasticity is important over the long run and that it is a first order parameter for medium range

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[1]Richard R. Nelson, "Aggregate Production Functions and Medium-Range Growth Projections", American Economic Review, Vol. LIV, (September, 1964), pp. 575-606.

[2]Marc Nerlove, "Recent Empirical Studies of CES and Related Production Functions", The Theory and Empirical Analysis of Production, Murray Brown, (National Bureau of Economic Research, New York, Columbia University Press, 1967), p. 57.

growth analysis.

Embodied Labor

Thus far I have considered only technical progress which is embodied in capital. We have in fact assumed that labor is homogeneous. However, economists have recognized the contribution of improved labor inputs to growth.

What form do these improvements in labor take? Schultz has suggested that the labor input is enriched through health facilities, on-the-job training, formal education, study programs and labor migration.[1] As with capital these improvements are embodied in the labor input and increase its productivity. The Schultz-Denison approach[2] may be contrasted to Arrow's.[3] All three recognize the contribution of knowledge and experience to growth. However, Denison embodies these factors in the labor input. Arrow uses gross investment as an index of experience. Unlike Denison, he specifies his function so that learning takes place in the capital goods industry only. No learning process can modify capital goods once they are built. We will follow the Denison method in our brief discussion.

Following the pattern of our definition of an efficiency unit of capital,

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[1] T. W. Schultz, "Investment in Human Capital", American Economic Review, Vol. LI, (March, 1961), pp. 1-18.

[2] E. F. Denison, "The Sources of Economic Growth and the Alternatives Before Us," Committee for Economic Development (New York: 1962).

[3] Kenneth Arrow, "The Economic Implications of Learning by Doing", Review of Economic Studies, Vol. XXIX, (June, 1961), pp. 155-172.

we may define an efficiency unit of labor. A man-hour performed in the year zero is taken as our standard unit. Then, assuming that the marginal rate of substitution between any man-hour and the standard unit is constant, a man-hour which may be perfectly substituted for x standard units represents x efficiency units of labor. Ignoring capital embodiment for the moment, technical progress may be defined in terms of improvements in each vintage of labor. The embodiment of labor may be symbolized as follows. If labor improves exponentially, then the labor input measured in man-hours (M) and the labor input measured in efficiency units (L) are related by the equation

3.12 $L = Me^{mt}$

Our Cobb-Douglas Production function may now be modified to include the embodiment aspects of both capital and labor.

$$Q(t) = D(t) \; g(J(t), M(t))$$

3.13 $Q(t) = e^{(m\alpha - \delta(1-\alpha))t} \, J(t)^{1-\alpha} \, M(t)^{\alpha}$

This model is fully embodied. That is, increases in output per capita which are not related to increased quantitative inputs are now attributed to either improvements in the quality of capital and/or labor. If we allow for disembodied progress the model will have the form

3.14 $Q(t) = De^{(m\alpha - \delta(1-\alpha))t} \, J(t)^{1-\alpha} M(t)^{\alpha}$

Using a similar formulation of the Cobb-Douglas function, Denison has shown that capital was responsible for 22.5% of the 2.93% annual increase in United States National Product between 1929 and 1957.[1] Increases in the quality and quantity of labor contributed 54%. This leaves a residual of

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[1]E. F. Denison, op cit, p. 103.

23.5% contributed by disembodied progress.

In order to apply this model we must know, or at least be able to estimate, the value of m (the improvement factor for labor). Neither of these alternatives is open to us at the present time. The aggregation problems presented by such a model are also forbidding. Fisher has shown that an equivalent stock of capital is defined if and only if technical change is purely capital embodied.[1]

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[1] F.M. Fisher, op cit, p. 265.

CHAPTER IV

EMBODIMENT vs DISEMBODIMENT AND NEUTRALITY vs NON-NEUTRALITY

Embodied vs Disembodied

The embodied form of the production function seems to have gained wider acceptance than the disembodied model. The ability to define technical progress more explicitly and the greater emphasis given to investment productivity generated enthusiasm for the disembodied representation which made time an explanatory variable in the growth process. If investment in capital ceased and the size of the labor force stagnated, then output would continue to grow. This growth is seemingly a result of the passage of time. Further, as mentioned before, increments in investment had a meagre impact on growth. The embodied model alleviated both of these difficulties. In its purely capital augmenting form, output growth requires that there be investment in capital goods. This implies that all productivity improvements are built into the capital stock. Not only did this give a more concrete meaning to technological progress, but the new model offered encouragement to investment policies. The modernizing mechanism strengthened the impact of investment on growth rates, at least in the short run. However, the embodied model generates a vicious cycle. Not only does technical progress increase the productivity of investment but it requires investment for its existence since new capital goods are required to execute the transition of innovation to technological change.

The embodied and disembodied models, as presented by Solow,

treat capital stock in a similar fashion. Both the net capital stock concept and the equivalent stock of capital allow for attrition due to physical wear and tear. The actual stock is given by

$$K(t) = \int I(v)\, e^{-\delta(v-t)}\, dv$$

whereas the equivalent stock is given by

$$J(t) = \int I(v)\, e^{\left(\frac{\lambda}{1-\alpha}+\delta\right) v}\, dv$$

The major distinction between the two models is in their treatment of the improvement factor. In the disembodied model $e^{\lambda t}$ is applied to labor and capital inputs and to all vintages in a uniform manner. Conversely, the embodiment approach applies the improvement factor only to capital inputs in a fashion which gives more recent vintages heavier weights.

Brown has shown that the two models yield identical rates of growth of output as a result of increased investment if businessmen exhibit faultless foresight.[1] Unlike Solow, his net capital stock concept allows for obsolescence as well as for depreciation. His equivalent stock of capital allows for depreciation only. Equivalence of the two models is shown as follows.

The embodied model is given by 3.8 and 3.9. If δ, physical deterioration, was non-existent and capital and labor inputs were held constant then 3.8 becomes (sic).

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[1]Murray Brown, op cit, p. 85.

$$4.1 \quad Q(t) = \overline{B}e^{\gamma t}$$

where $\overline{B}$ is a constant and $\gamma = \lambda$ (since $\delta = 0$)

Now the time path of output is unique, therefore it is given in the disembodied model by

$$4.2 \quad Q(t) = \overline{A}e^{\lambda t}$$

Referring back to 3.1, the time path of output would appear to be

$$4.3 \quad Q(t) = \overline{A}$$

Since 4.2 and 4.3 must be identical, the term A(t) in 3.1 must contain a trend term ($e^{\lambda t}$) to represent the true rate of growth of output. 3.1 becomes

$$4.4 \quad Q(t) = A_1(t)e^{\lambda t} f\left[L(t)\ K(t)\right]$$

Now if the rate of attrition of capital stock due to depreciation and obsolescence is symbolized by w then the capital stock is given by

$$4.5 \quad K(t) = \int_{-\infty}^{t} I(v)e^{w(v-t)}\,dv$$

Substituting this in 4.4 and letting α_1 represent elasticity of output with respect to labor the disembodied model becomes

$$4.6 \quad Q(t) = A_1(t)e^{\lambda t} L(t)^{\alpha_1}\left[\int_{-\infty}^{t} I(v)e^{w(v-t)}\,dv\right]^{1-\alpha_1}$$

Using this approach, Brown is able to show that the rate of growth of output due to an increment in investment is the same for both models provided $w = \gamma$ and $\alpha = \alpha_1$.[1,2]

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[1] Ibid.

[2] Empirical applications of the two models do not yield $\alpha = \alpha_1$. This is a result of the collinearity of the capital stock and time trend discussed in Chapter One. Further, the parameter γ refers to physical conditions. It relates the productivity of old and new capital. Conversely w is an economic parameter regerring to market values. Therefore the equivalence of γ and w is affected not only by statistical problems, but by market conditions as well.

If we concede, for the present, that the disembodied and embodied models yield identical results then the difference between the two models is solely one of interpretation. The improvement factor, $e^{\lambda t}$, in the disembodied model is in reality an improvement factor for the investment which replaces obsolete as well as depreciated capital. Consequently progress can no longer be termed disembodied or unembodied as Green prefers to call it.[1] However, disembodied change would seem to be a worthwhile concept. Not all change is built into capital. The truth undoubtedly lies somewhere between the two extremes. It seems that Brown has erred in attributing progress solely to changes in capital quality, just as the original Solow model over-emphasized the influence of disembodied change.

Brown's representation of the embodied model under conditions of constant labor inputs and constant investment is questionable. The embodied model is given bv:

$$4.7 \quad Q(t) = Be^{-\delta(1-\alpha)t} L(t)^{\alpha} \left[\int_{-\infty}^{t} e^{\left(\frac{\lambda}{1-\alpha} + \delta\right) v} I(v)\, dv \right]^{1-\alpha}$$

The conditions of his analysis are

$$4.8 \quad \delta = 0$$

$$4.9 \quad L(t) = L_o(t) \quad \text{a constant}$$

$$4.10 \quad I(t) = I_o(t) \quad \text{a constant}$$

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[1]H. A. John Green, "Embodied Progress, Investment, and Growth", American Economic Review, Vol. LI (March, 1961), pp. 138-150.

Substitution of these conditions into 4.7 yields

$$4.11 \quad Q(t) = B\, L_o(t)^{\alpha} \left[\int_{-\infty}^{t} e^{\frac{\lambda v}{1-\alpha}}\, I_o(v)\, dv \right]$$

Brown then proceeds to pull $I_o(v)$ out of the integral (because it is a constant) to arrive at 4.1

$$4.1 \quad Q(t) = B\, L_o(t)^{\alpha}\, I_o(t)^{1-\alpha} \int_{-\infty}^{t} e^{\lambda v}\, dv$$

$$Q(t) = \overline{B} e^{\lambda t}$$

In doing so Brown has violated the most important concept of the model. That is, investment in year t is more productive than investment in year t-1 as determined by the productivity index $e^{\lambda t}$ versus $e^{\lambda(t-1)}$ The fact that gross investment is constant certainly does not imply that the resultant capital stock is constant. Brown has converted the embodied to an disembodied model in which capital and labor are affected by the weighted improvement factor $e^{\lambda t}$.

Although the two models should yield identical results for the time path of output, the rates of growth of output that can be attributed to investment should not be equal. Investment productivity should be greater in the embodied model. It implies that all innovations must be built into the capital stock. The disembodied model does not require this. Rather than forcing the disembodied model to conform to the embodied representation, the ideal objective should be to determine the exact distribution of change.

Denison has asserted that the embodiment question is unimportant,

at least in an economy whose capital has a short life span.[1] Assuming complete embodiment, he has shown that a decrease of one year in the average age of capital over a ten year period increases the growth rate by only .16%.[2] This change is not particularly impressive in the light of two facts. First the average age of capital stock is relatively insensitive to changes in gross investment. Denison used Commerce Department figures for the United States to show that changes in the average age of the capital stock ranged from 0.3 years to 1.0 years between 1929 and 1961.[3] Second, Phelps has shown that decreases in the average age are not permanent. In the long run the average age of equipment will return to its equilibrium level.[4] Further, the effects of embodiment are exaggerated by the model which denotes a common improvement factor for all investment. Actually capital improvements are not uniform. There is a wide range of quality change in any given year. To apply a common rate to all capital is to overstate the effects of capital embodiment.

While the embodied model exaggerates the influence of new capital, the disembodied model underestimates it. The true model would seem to fall somewhere in between. Ideally the model should

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[1]Edward F. Denison, The Unimportance of the Embodied Question, American Economic Review, Vol. LIV, (March, 1964), pp. 90-93.

[2]Ibid, p. 90.

[3]Ibid, p. 91.

[4]Edmund S. Phelps, op cit, p. 559.

recognize capital augmenting and labor augmenting change. Further these inputs should be disaggregated to allow for varying degrees of productivity improvement. The application of such a model, however, is hindered not only by imposing data problems but also by compounded estimation difficulties.

Neutral and Non-Neutral Change

Discussion of both the embodied and disembodied models of technical progress has concentrated on neutral change. Because only one point on a production function can actually be observed it was deemed necessary to separate the effects of non-neutral change which shift the function. Recent literature has attempted to affix a measure to non-neutral change.

Brown and Popkin have set up a production function with allows them to disaggregate technical change.[1] Change is divided into (1) advances in neutral progress, (2) advances in non-neutral progress and (3) exploitation of economies to scale. They assume a Cobb-Douglas function of the form

$$4.12 \quad Q_{t_r} = A_r K_{t_r}^{a r} L_{t_r}^{b r} \; f(t_r) \, u_r$$

where t_r is a time index that has boundaries specified by the beginning

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[1] M. Brown and J. Popkin, "A Measure of Technological Change and Return to Scale", The Review of Economics and Statistics, Vol. XLIV (November, 1962), pp. 402-411.

and end of each period r. $f(t_r)$ measures neutral technical progress. $A_r \neq A_s$, $a_r \neq a_s$, $b_r \neq b_s$, $s \neq r$ is an indication of non-neutral changes. The degree of economies to scale is determined by the relationship $a_r + b_r \lesseqgtr 1$.[1]

Using Kendrick's non-farm domestic output 1890-1958 they fitted equation 4.12 to various time periods. 1890 to 1905 and 1935 to 1958 were chosen as base periods during which no non-neutral technological progress was assumed to occur. The method of tolerance intervals permitted the specification of epochs or periods in which there was no non-neutral change. They were able to determine three epochs 1890 to 1918, 1919 to 1937 and 1938 to 1958. The change in estimated parameters between epochs was considered non-neutral advance. Delineation of epochs required the rejection of the notion that technological progress is a slow process. Acceptance of well defined periods in which no non-neutral change requires a high rate of imitation. Thus, if all progress is embodied, investment rates must be high relative to the capital stock.

Brown and Popkin found that neutral change made the largest contribution to the growth of output per man-hour. This was followed by changes in inputs, non-neutral change and economies to scale.[2] Neutral change was found to be larger in the more recent epochs whereas non-neutral change described a cyclical pattern. Recent periods have exhibited

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[1] Ibid, p. 406.
[2] Ibid, p. 402

increasing returns, a result which agrees with the findings of Walters.

Non-neutral change would seem to be a comparatively small proportion of total productivity, possibly small enough to justify its exclusion in other studies.[1]

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[1] Ibid, p. 409. Contribution to output changes, United States, 1919-1958: neutral technological change (15.15%), non-neutral change (6.1%).

CHAPTER V

CONCLUSION

Summary

The Canadian results derived here for the embodied and disembodied models would seem to be consistent. They suggest that while disembodied change has been a factor in Canadian growth experience, capital augmenting change has been the dominant factor.

Results of similar studies of the Canadian economy are quite varied. Lithwick, Rymes and Post derived a figure of 2.4 for the rate of technological advance for the period 1949 to 1960.[1] Their study applies the disembodied Solow method to individual industries and then weights these to arrive at their figure for the whole economy. Holmes concluded that one-third of the average annual increase in Gross Domestic Product between 1941 and 1951 was due to technological progress.[2] Using the methods of Abramovitz, Holmes excluded agriculture, finance and services from his analysis to arrive at his result. Finally Domar realized a figure of 1.2 percent for the period 1949 to 1960.[3] Like Lithwick, Rymes and Post, Domar's investigation used the Solow method and covers the entire economy.

The figures for technical advance range from 2.4 per cent to the 1.192 per cent derived here. The 2.4 per cent figure realized by Lithwick,

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[1] Lithwick, Rymes and Post, op cit, p. 195.

[2] R.A. Holmes, "Factor Inputs, Technical Progress, and Economic Growth in Canada", The Western Economic Journal, vol IV (Summer, 1966), pp. 247 - 261.

[3] E.D. Domar, S.M. Eddre, G.H. Herrick, P.M. Hohenberg, M.D. Intriligator, I. Miyamoto, "Economic Growth and Productivity in the U.S., Canada, U.K., Germany and Japan in the Post-War Period", Review of Economics and Statistics, vol. XLVI (February, 1964), pp. 33 - 41.

Rymes amd Post compares with Domar's 2.4 per cent for the period 1949 to 1956. Inclusion of the late fifties in Domar's study would seem to decrease the contribution of technical admance. It is highly likely that the trend of the late fifties would have prevailed in the early sixties. Certainly there were nò major innovations to suggest otherwise. This would partially explain the divergence between Lithwick's results and that derived here. While Holme's figure exceeds the 1.192 developed herein, the deviation is not so drastic. The divergence is probably a product of time periods (1941 to 1951 vs 1947 to 1964) and method (Abramovitz vs Solow).

Policy Recommendations

The results of this thesis indicate that Canadian economic growth can be expedited by increased investment in capital. This agrees with Lithwick's conclusions.[1] He found that higher levels of capital per worker in the United States were responsible for the Canada - United States gap in output per worker. However, while United States capital per worker estimates were found to exceed Canada's on an economy wide basis, he noted that capital intensity is 29 per cent higher in Canadian manufacturing.[2] Yet productivity in Canadian manufacturing lags its American counterpart by 29.8 per cent. This observation colors our policy recommendation. We must augment our capital stock but several other simultaneous improvements must be made. Investment in research and

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[1] Lithwick, Rymes and Post, op cit, p. 9.

[2] Ibid, p. 10

and development must be encouraged. Not only will this result in original improvements in technology, but the technology which filters across the border can be made more suitable for the smaller scale Canadian industry. Further, inefficiencies which have persisted in Canada due to our protectionist policies must be removed. Finally, the fact that high levels of capital investment have not deterred an increasing productivity gap leads us to become suspicious of the quality of our labor force. The economic foundations for wage parity demands become very questionable in the light of such productivity comparisons.

The recommendations noted above must be regarded as weak in light of the inadequacies of production function analysis as a policy tool. Two considerations are pertinent. First, there are the uncertainties associated with aggregate production theory; the arguments over aggregation, the statistical problem of identification and the basic question of the importance of embodiment. Second, there are the weak links which exist between the economic mechanisms portrayed by the production function and the actual policies which influence them. For example, consider investment. The importance of investment to growth is indicated by the embodied model. Yet the precise quantitative impact of future investment cannot be derived. We can only use our historical results as guidelines. If we conclude that investment is significant to growth and that growth is desirable *per se*, then how can policy

influence investment? Economic theory has given us several variables to play with; interest rates, tax credits, profit rates, etc. But here again we are faced with the problem of knowing the probable direction of impact but not the magnitude.

The policy benefits of production function analysis lie in the ability to indicate the historical importance of certain variables and the direction of their influence. This is at least a starting point. Nelson has suggested that because of the ignorance associated with precise magnitudes, policy making becomes a sequential decision making process.[1] In this light production function analysis becomes a useful tool, enabling economists to utilize historical data to determine the direction and approximate magnitude of policy induced change under varying economic circumstances. Economists are then in a position to evaluate economic programs. Should they be modified or should they be discontinued? Production functions become representations of the constraints to which decision rules are applied.

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[1] Richard R. Nelson, "Aggregative Production Functions and Economic Growth Policy", The Theory and Empirical Analysis of Production, Murray Brown ed. (National Bureau of Economic Research, Studies in Income and Wealth, No. 31, New York, Columbia University Press, 1967).

APPENDIX

Figure 1a

THE PRODUCTION SURFACE

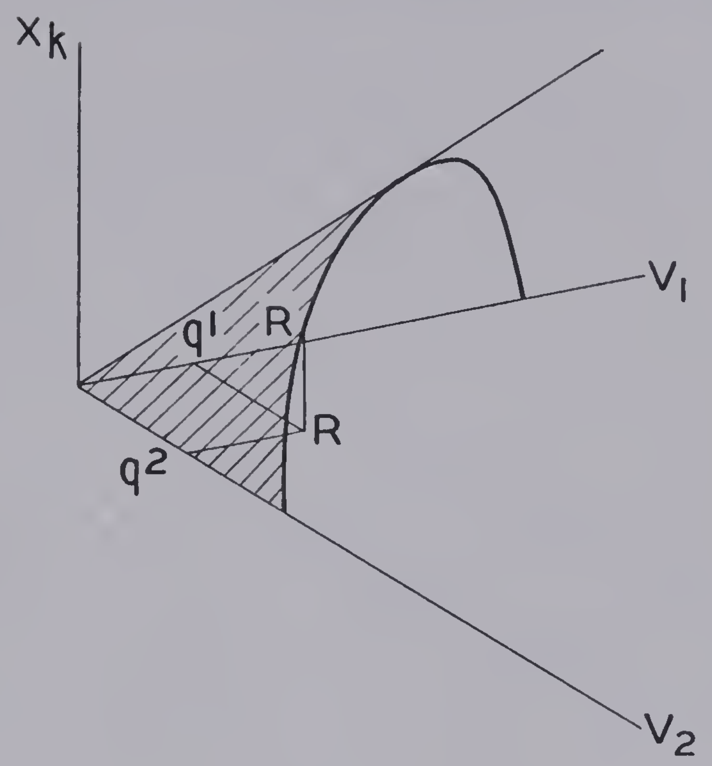

Figure 1b

THE TWO DIMENSIONAL PRODUCTION SURFACE

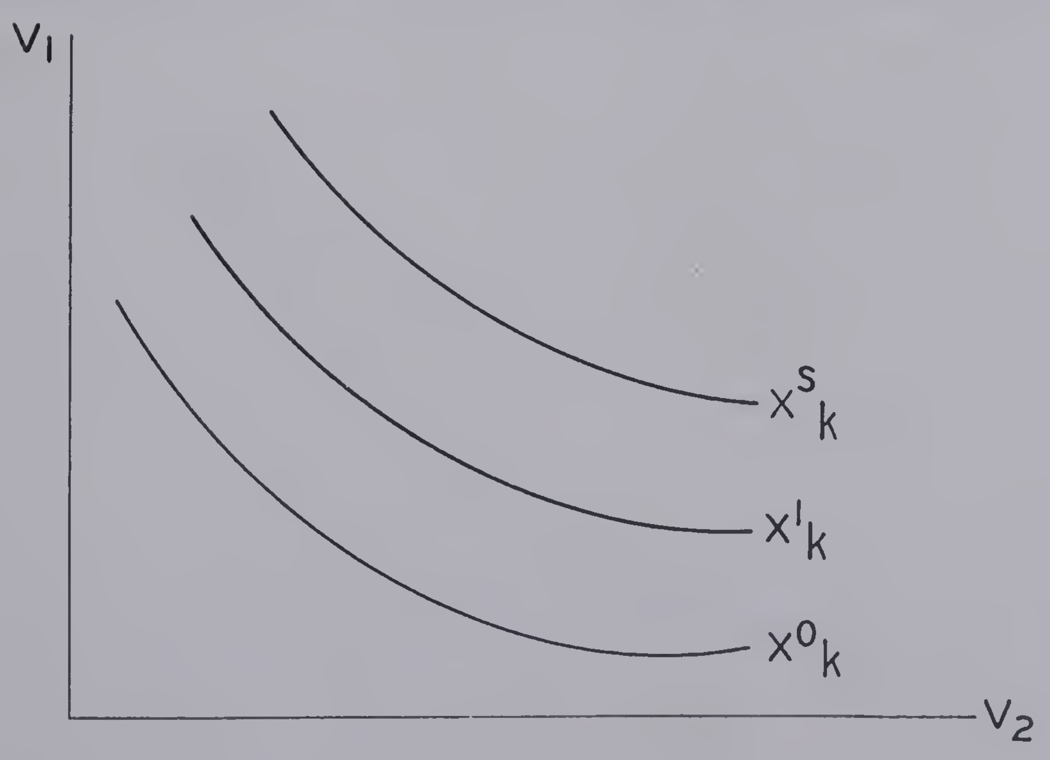

Figure 2a

NEUTRAL TECHNOLOGICAL CHANGE

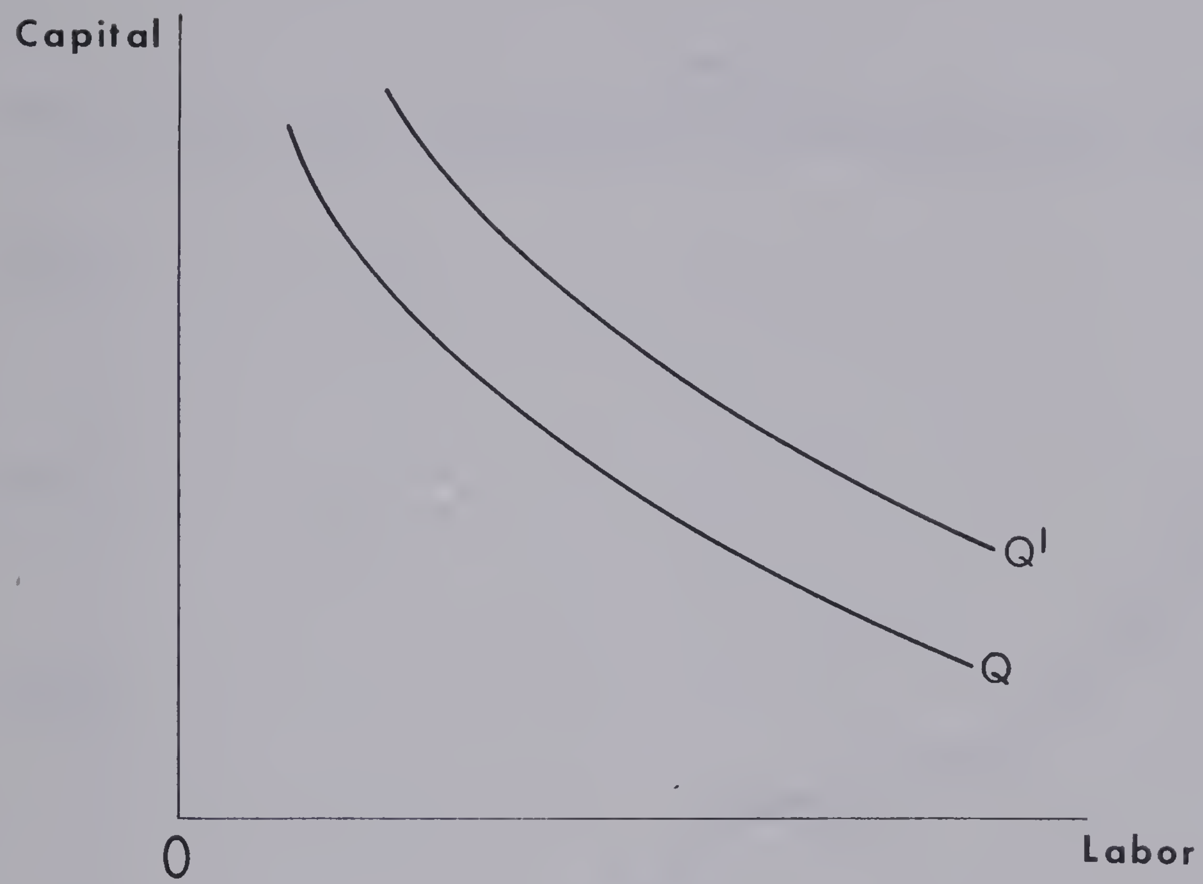

Figure 2b

NON-NEUTRAL TECHNOLOGICAL CHANGE

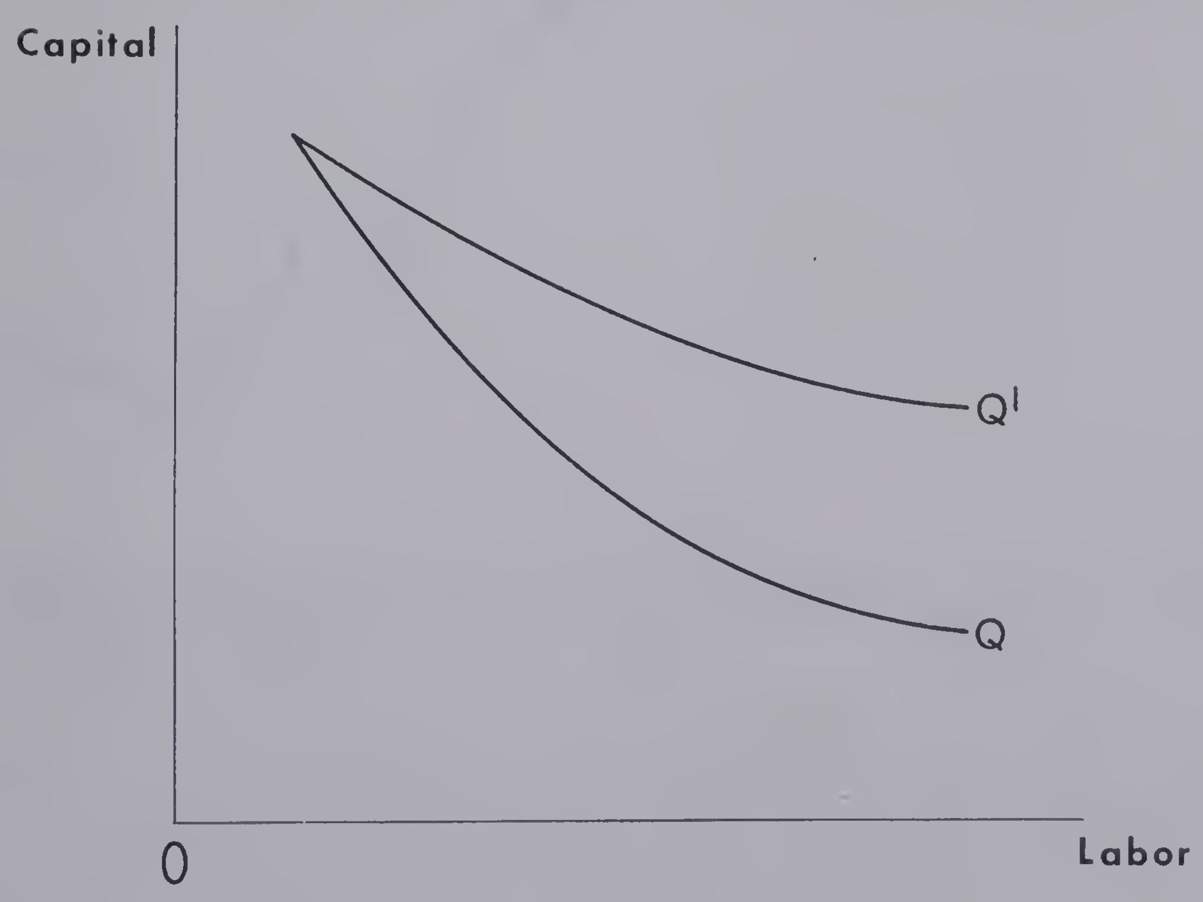

Figure 3

TIME PATH OF CANADIAN TECHNOLOGICAL CHANGE

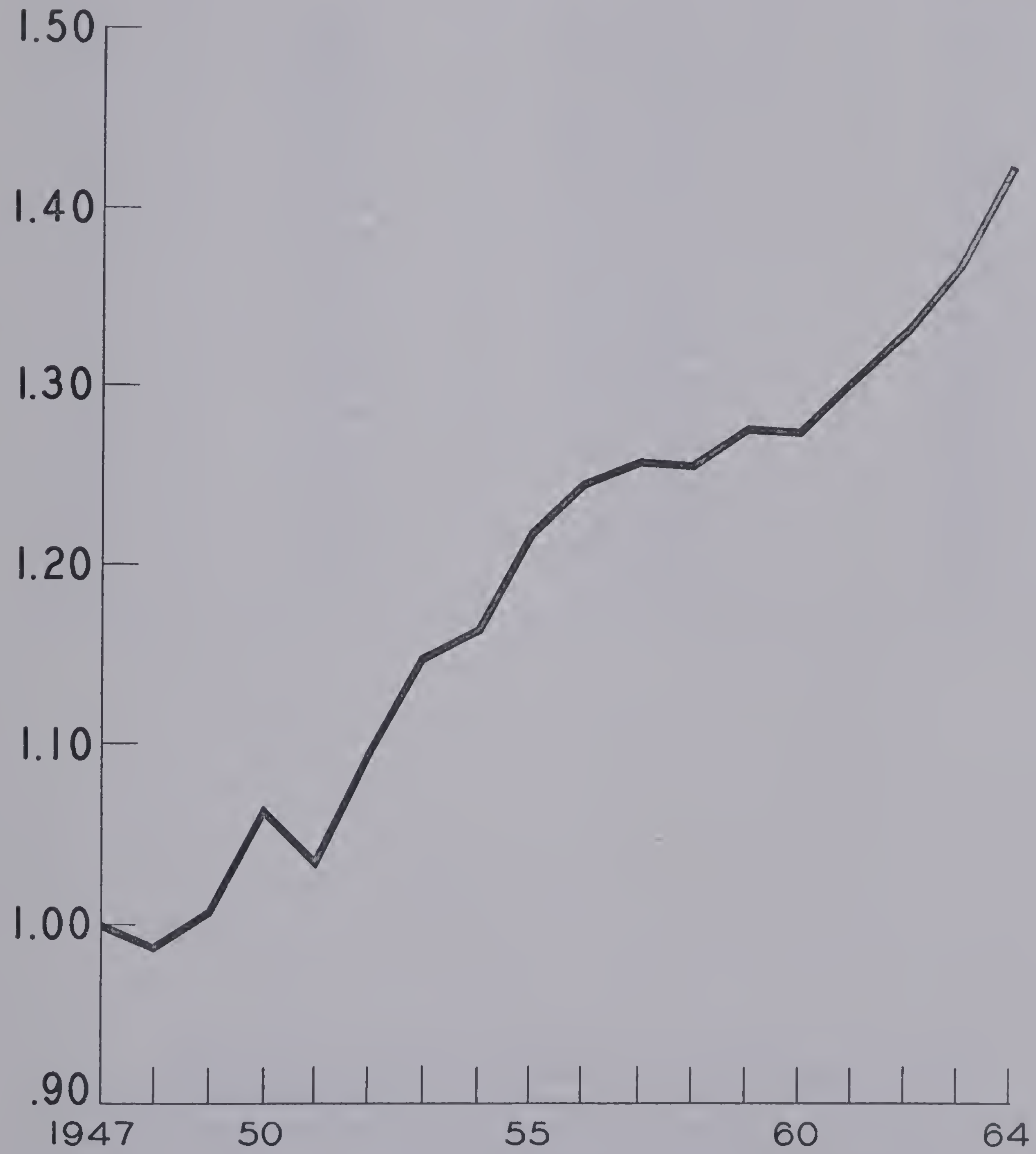

TABLE I

PRIVATE NON-AGRICULTURAL OUTPUT PER MAN-HOUR
(Millions of 1957 Dollars)

Year (1)	Gross National Expenditure [a] (2)	Agricultural Income [b] (3)	Government Expenditure on Goods and Services [c] (4)
1947	20439	1739.1305	2762
1948	20821	1898.0716	2839
1949	21626	1650.7936	3175
1950	23114	1703.6083	3349
1951	24531	2239.8610	4188
1952	26514	2164.6409	5250
1953	27525	1732.6733	5251
1954	26714	1083.0671	5098
1955	29018	1351.8716	5319
1956	31508	1493.3059	5664
1957	31909	1026.0000	5722
1958	32284	1188.1188	6113
1959	33398	1072.7273	6205
1960	34200	1118.8679	6268
1961	35081	944.7048	6562
1962	37429	1380.0738	6811
1963	39352	1560.2901	6848
1964	41895	1297.8724	7122

[a] National Accounts, Income and Expenditure, Annuals, Table 18, Line 20.

[b] Industrial Distribution of Gross Domestic Product at Factor Cost, D.B.S. Work Sheets, deflated by Implicit G.N.E. Deflator (1957=100).

[c] National Accounts, Income and Expenditure, Annual, Table 18, Line 6.

TABLE I (Cont'd)

PRIVATE NON-AGRICULTURAL OUTPUT PER MAN-HOUR
(Millions of 1957 Dollars)

Year	Private Non-Agricultural Gross National Expenditure[d] (5)	Private Non-Agricultural Man-Hours[e] (6)	Private Non-Agricultural GNE per Man-Hours[f] (7)
1947	15937.8694	6.4399	2474.8635
1948	16083.9283	6.6009	2436.6254
1949	16800.2060	6.7949	2472.4893
1950	18061.3920	6.9339	2604.7926
1951	18103.1390	7.1465	2533.1470
1952	19099.3590	7.3782	2588.6033
1953	20541.3270	7.5643	2715.5629
1954	20532.9330	7.4104	2770.8325
1955	22347.1280	7.7637	2878.3942
1956	24350.6950	8.3181	2927.4648
1957	25161.0000	8.5093	2956.8662
1958	24982.8820	8.4349	2961.8417
1959	26120.2730	8.6602	3016.1115
1960	26813.1320	8.7801	3053.8605
1961	27574.2960	8.7561	3149.1462
1962	29237.9260	9.1044	3211.3927
1963	30943.7100	9.3893	3295.6277
1964	33475.1280	9.7530	3432.2739

[d] (2) - (3) - (4)

[e] Total number of hours worked per worker in Non-Agricultural Sector (Econometric Unit, Department of Finance).

[f] Number of paid workers in Private Non-Agricultural Sector (Econometric Unit, Department of Finance), (4)/(5).

TABLE II

NET CAPITAL STOCK UTILIZED
(Millions of 1957 Dollars)

Year (1)	Net Stock of Machinery and Equipment (2)	Net Stock of Non-Residential Construction (3)	Total Net Capital Stock [a] (4)	Employment Rate [b] (5)	Utilized Capital Stock [c] (6)
1947	6605.8855	13276.7345	19882.6200	.9777	19440.0690
1948	6722.6714	13454.0269	20176.6980	.9772	19717.5850
1949	6850.1236	13655.3009	20505.4250	.9715	19921.4570
1950	6981.4754	13879.0800	20860.5550	.9639	20107.9590
1951	7160.7706	14132.8800	21293.6510	.9758	20778.9420
1952	7372.2729	14442.9804	21815.2540	.9687	21131.7010
1953	7608.1613	14775.9485	22384.1100	.9699	21711.5430
1954	7758.5660	15093.5863	22852.1530	.9545	21811.4800
1955	7920.1947	15442.4934	23362.6890	.9563	22340.6290
1956	8217.3870	15952.9264	24170.314	.9659	23346.5246
1957	8509.6592	16567.0560	25076.715	.9538	23918.4930
1958	8665.8507	17091.2710	25757.1210	.9297	23946.1560
1959	8844.4436	17536.9130	26381.3560	.9405	24810.8030
1960	9026.0015	17960.7630	26986.7650	.9304	25110.0530
1961	9148.1703	18402.8270	27550.9980	.9285	25580.4780
1962	9303.3642	18816.6260	28119.9900	.9410	26460.1010
1963	9501.2432	19249.7920	20751.0360	.9446	27159.3780
1964	9807.0061	19762.4500	29569.4560	.9532	28185.8420

[a] (2) + (3)

[b] Total Employed (Thousands) D.B.S. Catalogue 71-001/Total Civilian Labour Force (Thousands) D.B.S. Catalogue 71-007.

[c] (4) x (5).

TABLE III

CAPITAL'S SHARE IN NATIONAL INCOME[a]

Year	Corporation Profits[b]	Dividends to Non-residents	Rent, Interest and Misc. Investment Income	Non-farm Unincorporated Enterprise Income
(1)	(2)	(3)	(4)	(5)
1947	1814	-248	519	587
1948	1964	-249	651	635
1949	1879	-317	703	695
1950	2522	-404	890	715
1951	2825	-370	1020	760
1952	2698	-334	1175	786
1953	2611	-317	1329	844
1954	2290	-327	1511	829
1955	2965	-395	1684	896
1956	3345	-437	1767	983
1957	3056	-475	1980	1004
1958	3075	-470	2104	1063
1959	3498	-501	2281	1096
1960	3277	-470	2390	1095
1961	3438	-588	2529	1125
1962	3819	-584	2832	1200
1963	4188	-614	3078	1276
1964	4819	-713	3262	1360

[a]National Income and Expenditure by Quarters, D.B.S. 13-519, Table 1.
[b]Before taxes and before dividends paid to non-residents.

TABLE III (Cont'd)

CAPITAL'S SHARE IN NATIONAL INCOME

Year	Inventory Valuation Adjustment (6)	Capital Consumption Allowance (7)	Capital's Absolute Share [c] (8)	Capital's Relative Share [d] (9)
1947	-571	1223	3223	0.245
1948	-506	1441	3935	0.260
1949	-112	1673	4520	0.277
1950	-374	1913	5171	0.287
1951	-643	2203	5794	0.274
1952	106	2422	6907	0.288
1953	- 11	2673	7118	0.284
1954	86	2905	7294	0.293
1955	-189	3266	8226	0.303
1956	-238	3642	9064	0.296
1957	- 78	4009	10446	0.343
1958	- 35	3899	9635	0.293
1959	-130	4154	10399	0.299
1960	- 55	4293	10530	0.293
1961	- 86	4349	10766	0.292
1962	-130	4892	12030	0.296
1963	-200	5198	12925	0.298
1964	-131	5600	14197	0.300

[c] (2) + (3) + (4) + (5) + (6) + (7) = (8)
[d] (8) ÷ Gross National Expenditure, current dollars.

TABLE IV

DISEMBODIED TECHNICAL CHANGE

Year	$\left[\frac{K}{L} - (\frac{K}{L})_{t-1}\right] \Big/ \frac{K}{L}$ [a]	W_K [b] $\times \left[\frac{K}{L} - (\frac{K}{K})_{t-1}\right] \Big/ \frac{K}{L}$	$\left[\frac{Q}{L} - (\frac{Q}{L})_{t-1}\right] \Big/ \frac{Q}{L}$ [c]
(1)	(2)	(3)	(4)
1947			
1948	-0.0105	-0.0026	-0.0154
1949	-0.0185	-0.0048	0.1472
1950	-0.0109	-0.0030	0.1535
1951	0.0026	-0.0008	-0.2751
1952	-0.0150	-0.0041	0.0219
1953	0.0022	0.0062	0.0491
1954	0.0255	0.0072	0.0204
1955	-0.0224	-0.0066	-0.0388
1956	-0.0246	-0.0075	0.0170
1957	0.0015	0.0004	0.0100
1958	0.0100	0.0034	0.0017
1959	0.0091	0.0027	0.0183
1960	-0.0018	-0.0005	0.1251
1961	0.0215	0.0063	0.0312
1962	-0.0052	-0.0015	0.0198
1963	-0.0047	-0.0014	0.0262
1964	-0.0009	-0.0003	0.0415

[a] Rate of change of utilized capital per man-hour.
[b] Relative share of capital.
[c] Rate of change of private non-farm G. N. E. per man-hour.

TABLE IV (Cont'd)

DISEMBODIED TECHNICAL CHANGE

Year	$1 + \frac{\Delta A(t)}{A(t)}$ [d] (5)	$A(t) \left[1 + \frac{\Delta A(t)}{A(t)} \right]$ [e] (6)
1947		1.0000
1948	.9871	.9871
1949	1.0195	1.0064
1950	1.0565	1.0633
1951	.9717	1.0332
1952	1.0260	1.0952
1953	1.0484	1.1482
1954	1.0131	1.1633
1955	1.0454	1.2161
1956	1.0245	1.2453
1957	1.0096	1.2579
1958	.9983	1.2557
1959	1.0156	1.2752
1960	1.0130	1.2735
1961	1.0249	1.3052
1962	1.0213	1.3330
1963	1.0276	1.3698
1964	1.0417	1.4270

[d] (4) - (3) = (5)
[e] $(6)_t \times (5)_{t+1} = (6)_{t+1}$

TABLE V

EMBODIED TECHNICAL CHANGE
(Millions of 1957 Dollars)

Year (1)	L^a (2)	Q^b (3)	Business Gross Fixed Capital Formation (I)[c] (4)	$\text{Log}\left[\frac{R-R_{t-1}}{R} + 0.04\,R/I\right]^d$ (5)	$\text{Log}\left[\frac{B-B_{t-1}}{B} + 0.04\,B/I\right]^e$ (6)
1947	41.4723	15937.8694	894.00	7.4773	11.7644
1948	43.5719	16083.9283	989.00	7.9720	12.3039
1949	46.1701	16800.2060	1087.50	8.2183	12.5755
1950	48.0790	18061.3920	1135.75	7.1043	11.4153
1951	51.0725	18103.1390	1184.00	8.0949	12.4838
1952	54.4386	19099.3590	1286.50	8.2695	12.6846
1953	57.2186	20541.3270	1407.00	7.6855	11.9951
1954	54.9138	20532.9330	1335.00	8.3742	12.8333
1955	60.2758	22347.1280	1420.50	8.3989	12.9057
1956	69.1894	24350.6950	1753.50	8.0689	12.5704
1957	72.4090	25161.0000	1833.75	7.4491	11.7227
1958	71.1478	24982.8820	1706.25	8.2199	12.7417
1959	74.9999	26120.2730	1640.50	8.1276	12.6465
1960	77.0898	26813.1320	1557.75	8.2649	12.7857
1961	76.6696	27574.2960	1553.25	8.4649	13.0424
1962	82.8908	29237.9260	1563.50	8.5220	13.1196
1963	88.1594	30943.7100	1654.50	8.6934	13.3255
1964	95.1219	33475.1280	1901.50		

[a]Man-hours worked per non-agricultural worker times number of employed paid non-agricultural workers, <u>Special Surveys</u>, D. B. S., Confidential.

[b]Private non-farm G. N. E.

[c]<u>National Accounts, Income and Expenditure</u>, Annuals, Table 18, Line 28.

[d]$R = Q^3/L^2$, i.e. $= 2/3$

[e]$B = Q^4/L^3$, i.e. $= 3/4$

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